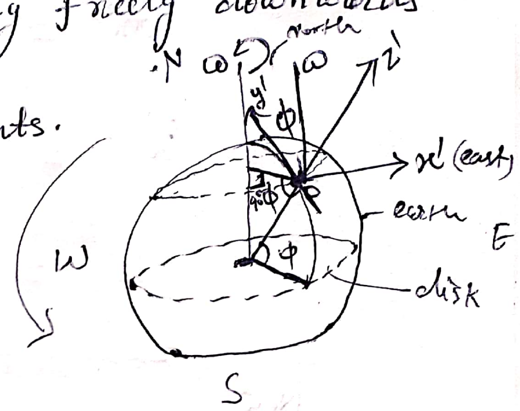
Effect of Coriolis force on bodies falling freely downwards on earth :->



The angular velocity have two components.

ie $\quad \vec{\omega} = \omega \cos\phi \, \hat{J}' + \omega \sin\phi \, \hat{K}'$

Coriolis force $\quad F_c = -2m(\vec{\omega} \times \vec{v}_{s'})$

$$m\vec{a}_c = -2m(\vec{\omega} \times \vec{v}_{s'})$$

$$\vec{a}_c = -2(\vec{\omega} \times \vec{v}_{s'})$$

(earth is rotating from west to east)

if $\vec{v}_{s'}$ be the velocity acquired by the body in time (t) taken by it to fall through a height (h) we have

$$\vec{v}_{s'} = -v\,\hat{K}' \quad \text{(velocity directed downwards)}$$

Now

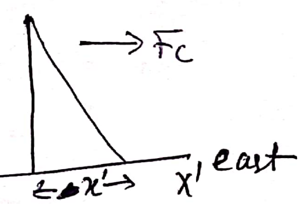
$$\vec{a}_c = -2(\vec{\omega} \times \vec{v}_{s'})$$

$$\vec{a}_c = -2\left[(\omega\cos\phi\,\hat{J}' + \omega\sin\phi\,\hat{K}') \times v(-\hat{K}')\right]$$

$$\vec{a}_c = 2\omega v(\cos\phi\,\hat{i}' + 0)$$

$$\vec{a}_c = 2\omega v\cos\phi\,\hat{i}'$$

$$\vec{F}_c = 2m\omega v\cos\phi\,\hat{i}'$$



Now we have to calculate x' deflection

$$\vec{a}_c = 2\omega v\cos\phi\,\hat{i}'$$

$$\frac{d^2\vec{x}'}{dt^2} = 2\omega v\cos\phi\,\hat{i}'$$

for vertical motion $\quad v = gt \quad$ (if start from rest)

$$\frac{d}{dt}\left(\frac{d\vec{x}'}{dt}\right) = 2\omega g t\cos\phi\,\hat{i}'$$

$$\int \frac{d}{dt}\left(\frac{d\vec{x}'}{dt}\right) = \int 2\omega g\cos\phi\, t\, dt\,\hat{i}'$$

$$\frac{d\vec{x}'}{dt} = \cancel{2}\omega g\cos\phi\,\frac{t^2}{\cancel{2}}\,\hat{i}'$$

$$\int d\vec{x}' = \int \omega g \cos\phi \, t^2 \, dt \, \hat{i}'$$

$$\vec{x}' = \omega g \cos\phi \frac{t^3}{3} \hat{i}'$$

$$\boxed{x' = \omega g \cos\phi \frac{t^3}{3}}$$

Now

$$h = ut + \frac{1}{2} g t^2$$

$$u = 0$$

$$h = \frac{1}{2} g t^2$$

$$\boxed{t = \sqrt{\frac{2h}{g}}}$$

$$x' = \frac{1}{3}\omega g \cos\phi \left(\frac{2h}{g}\right)^{3/2}$$

$$x' = \frac{1}{3} \omega \cancel{g} \cos\phi \frac{2h}{\cancel{g}} \sqrt{\frac{2h}{g}}$$

$$\boxed{x' = \frac{2}{3} \omega h \cos\phi \sqrt{\frac{2h}{g}}}$$

$$x' = \sqrt{\frac{8}{9g}} \, h^{3/2} \omega \cos\phi$$

$$x' = \left(\frac{8}{9g}\right)^{1/2} h^{3/2} \omega \cos\phi$$

at the equator $\phi = 0$, $\cos\phi = 1$

$$x' = \left(\frac{8}{9g}\right)^{1/2} h^{3/2} \omega \quad \text{(maximum)}$$

(deflected towards right)

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Effect of Coriolis force on a particle moving horizontally on the earth:-

we know that

$$\vec{\omega} = \omega\cos\phi\,\hat{j}' + \omega\sin\phi\,\hat{k}'$$

and $$\vec{v}_{s'} = v_{x'}\hat{i}' + v_{y'}\hat{j}'$$ (z component is not there b/c of horizontal motion)

Now Coriolis Force

$$\vec{F}_c = -2m(\vec{\omega}\times\vec{v}_{s'})$$

$$\vec{F}_c = -2m\begin{vmatrix} \hat{i}' & \hat{j}' & \hat{k}' \\ 0 & \omega\cos\phi & \omega\sin\phi \\ v_{x'} & v_{y'} & 0 \end{vmatrix}$$

$$\vec{F}_c = -2m\left[\hat{i}'(-\omega v_{y'}\sin\phi) - \hat{j}'(-\omega v_{x'}\sin\phi) + \hat{k}'(-\omega v_{x'}\cos\phi)\right]$$

$$\vec{F}_c = 2m\left[\omega v_{y'}\sin\phi\,\hat{i}' - \omega v_{x'}\sin\phi\,\hat{j}' + \omega v_{x'}\cos\phi\,\hat{k}'\right]$$

$$\vec{F}_c = \underbrace{2m\omega(v_{y'}\sin\phi\,\hat{i}' - v_{x'}\sin\phi\,\hat{j}')}_{\text{Horizontal part}} + \underbrace{2m\omega v_{x'}\cos\phi\,\hat{k}'}_{\text{Vertical part}}$$

$$\vec{F}_{c(H)} = 2m\omega\sin\phi\,(v_{y'}\hat{i}' - v_{x'}\hat{j}')$$

$$\vec{F}_{c(V)} = 2m\omega v_{x'}\cos\phi\,\hat{k}'$$

$$|\vec{F}_{c(H)}| = 2m\omega\sin\phi\sqrt{v_{y'}^2 + v_{x'}^2}$$

$$|\vec{F}_{c(V)}| = 2m\omega v_{x'}\cos\phi$$